

Past Eras In Cyclic Cosmological Models

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Abstract

In infinitely cyclic cosmology past eras are discussed using set theory and transfinite numbers. One consistent scenario, already in the literature, is where there is always a countably infinite number, \aleph_0 , of universes and no big bang. I describe here an alternative where the present number of universes is \aleph_0 and in the infinite past there was only a finite number of universes. In this alternative model it is also possible that there was no big bang.

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Introduction.

It was suggested already in the first half of the twentieth century, most emphatically by Tolman [1], that the initial singularity of a big bang cosmology might be avoided by hypothesizing a cyclic universe in which the expansion era ends at a turnaround to a contraction era which itself ends in a bounce.

Implementation of cyclic cosmology confronted, however, a seemingly impossible problem which we shall call the *Tolman conundrum*. The Tolman conundrum is that, because entropy monotonically increases, future cycles are larger and longer while in the past cycles were smaller and shorter leading at a finite past time to a big bang. The motivation of avoiding a big bang was thus frustrated.

It is fair to say that the Tolman conundrum and the failure to solve it led to the continued acceptance of the big bang from Tolman's time until the twenty-first century. Incidentally it is worth remarking that in his work Tolman never found it necessary to assume the possibility of more than one universe.

A major observational discovery in cosmology was of the accelerated expansion rate of the universe and the concomitant dark energy. The idea that dark energy might aid in constructing a consistent cyclic universe was pioneered in a useful and important series of papers by Steinhardt and Turok [2]. These authors conceived of the idea that branes colliding in an extra space dimension could provide an alternative to inflationary cosmology and, beyond that, underly a cyclic cosmology. These papers contain a large number of important new ideas and make considerable progress towards avoiding a big bang. They do not, however, solve the Tolman conundrum.

In a different approach ^{#2} Baum and the author [3] solved the Tolman conundrum while disregarding other obstacles in the hope that they could be solved if the Tolman conundrum is.

The resultant model leads to some unexpected but logically inevitable assumptions needed to reduce the dimensionless entropy of the universe from over one googol (10^{100}) to zero at turnaround.

These are those assumptions:

- (i) The dark energy must have equation of state $w = p/\rho < -1$, with $|(w + 1)|$ arbitrarily small. This ensures approach to a big rip and possible fragmentation at turnaround into more than one googol of causally disconnected patches.

^{#2}Other recent works on cyclic models are listed in [4]

(ii) The universe contracts adiabatically with zero entropy, empty of matter and containing only dark energy. This avoids the otherwise necessary fine tuning of initial conditions at the beginning of the expansion era to less than a part in one googolplex ($10^{10^{100}}$).

In the model of [2] a difficult technical problem concerns the singularity at the bounce. In the complementary model of [3] it is the turnaround that requires as an equally challenging problem the computation of entropy in extreme spacetime backgrounds.

The number of universes

In the model of [3] the following relationship between the number of universes Σ_n after cycle n and Σ_{n+1} after cycle $(n + 1)$ occurs

$$\Sigma_{n+1} = N\Sigma_n \tag{1}$$

where N is a finite number necessarily bigger than one googol. One may take *e.g.* $N = 10^{123}$ as the number of universes spawned at each turnaround.

Let me label the present expansion era as $n = 0$. It is straightforward to see that to avoid a big bang the present number Σ_0 must be infinite. I therefore set it equal to the smallest infinite ordinal $\Sigma_0 = \aleph_0$.

Higher transfinite numbers than \aleph_0 while possible to define precisely are more difficult to describe [5-7]. They are surely irrelevant to cosmology.

With these preliminaries I can now define two specific cyclic models which I shall represent by exhibiting time-ordered sets S_1 and S_2 of their respective transfinite sequences of cardinal Σ_n .

The first model

S_1 will characterize my first model and S_2 my second.

I first note that multiplication of \aleph_0 by any finite number leaves it unchanged as \aleph_0 , for example

$$N^p \aleph_0 \equiv \aleph_0 \tag{2}$$

for all finite p .

I introduce the notation that a double ellipsis .. denotes a finite number of elements while a triple ellipsis ... denotes a countably infinite number (\aleph_0) of elements.

I am now ready to define the first cyclic model by the transfinite sequence S_1 .

$$S_1 = [..., \aleph_0, \aleph_0, \aleph_0, ...] \quad (3)$$

This implies that the model always contains an infinite number \aleph_0 of universes^{#3}. It was the only model discussed in the second paper of [3]. The transfinite sequence S_1 is equivalent to its own anti-time-ordering S_1^* and I can therefore write: $S_1 \equiv S_1^*$.

At first sight, S_1 seems the only consistent possibility. However, I believe there exists an alternative cyclic model characterized by a strictly time-ordered transfinite sequence S_2 .

The alternative model

Can a cyclic model exist which begins with a finite number of universes, for example one universe?

I have reached a positive conclusion by considering a time-ordered transfinite sequence of cardinals S_2 with $S_2 \neq S_2^*$ as follows

$$S_2 = [1, N, N^2, ..., N^p, N^{p+1}, N^{p+2}, ..., \aleph_0, \aleph_0, \aleph_0, ...] \quad (4)$$

where p , like N , is finite and we employ Eq.(2).

In the transfinite sequence S_2 the first infinite element \aleph_0 must have absence of a precedent. We hypothesize that S_2 corresponds to a cyclic cosmology. Provided that at the present time $t = t_0$ there is an infinite number \aleph_0 of universes there was no big bang.

This introduces an interesting concept from mathematics because while physically in the model [3] it is puzzling if a finite number of universes can spawn an infinite number yet this is expressed mathematically by the useful notion *absence of precedent* which was unnecessary in S_1 .

This is just as in set theory the first transfinite ordinal coming after 1, 2, 3, ... is \aleph_0 which has a following transfinite $\aleph_0 + 1$ but \aleph_0 has no precedent.

^{#3}Because entropy is constant at \aleph_0 I can say that this cyclic model eliminates an arrow of time.

All that is necessary to eliminate a big bang in Eq.(4) is that the present universe correspond to a cardinal of S_2 preceded by a triple ellipsis.

These considerations are applicable to all cyclic models [2–4] which solve the Tolman conundrum.

Discussion

The distinction between the first and the alternate model is that in the alternate model we both avoid a big bang and allow the original creation to be of a finite number of universes possibly only one universe. One can say that it is easier to create only one universe as in the alternate model than go to the bother of creating an infinite number \aleph_0 of universes as in the first model.

Both models are characterized by a present equation of state of dark energy satisfying $w < -1$ as will be tested by the Planck mission and subsequent observations.

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References

- [1] R.C. Tolman, *On the Theoretical Requirements for a Periodic Behaviour of the Universe*. Phys. Rev. **38** 1758 (1931). See also: *Relativity, Thermodynamics and Cosmology*. Oxford University Press. (1934).
- [2] P.J. Steinhardt and N. Turok, *A Cyclic Model of the Universe*. Science **296**, 1436 (2002);
P.J. Steinhardt and N. Turok, *Cosmic Evolution in a Cyclic Universe*. Phys. Rev. **D65**, 126003 (2002). [hep-th/0111098](#);
L.A. Boyle, P.J. Steinhardt and N. Turok, *A New Duality Relating Perturbations in Expanding and Contracting Friedmann Cosmologies*. Phys Rev. **D70**, 023504 (2004) [hep-th/0403026](#);
P.J. Steinhardt and N. Turok, *Why the Cosmological Constant is Small and Positive*. Science, **312**, 1180 (2006). [astro-ph/0605173](#).
- [3] L. Baum and P.H. Frampton, *Turnaround in Cyclic Cosmology*. Phys. Rev. Lett. **98**, 071301 (2007). [hep-th/0610213](#);
P.H. Frampton, *Cyclic Universe and Infinite Past*. Mod. Phys. Lett. **A22**, 2587 (2007). [arXiv:0705.2730 \[astro-ph\]](#).
- [4] B. McInnes, *Arrow of Time in String Theory*. Nucl. Phys. **B782**, 1 (2007). [hep-th/0611088](#).
M.P. Dabrowski, T. Denkiewicz and M.A. Hendry, *How far is it to a Sudden Future Singularity of Pressure?*. Phys. Rev. **D75**, 123524 (2007). [arXiv: 0704.1383 \[astro-ph\]](#).
A. Tziolas and A. Wang, *Colliding Branes and Formation of Spacetime Singularities*. Phys. Lett. **B661**, 5 (2008). [0704.1311 \[hep-th\]](#).
E.N. Saridakis, *Cyclic Universes from General Collisionless Braneworld Models*. Nucl. Phys. **B808**, 224 (2009). [arXiv: 0710.5269 \[hep-th\]](#).
M. Novello and S.E. Perez Bergliaffa, *Bouncing Cosmologies*. Physics Reports **463**, 127-213 (2008). [arXiv:0802.1634 \[astro-ph\]](#).

- [5] G. Cantor, *Contributions to the Founding of the Theory of Transfinite Numbers*. Translator: P.E.B. Jourdain. Dover Publications, Inc. New York (1952).
- [6] H.W. Woodin, *The Continuum Hypothesis, Part I*. Notices of the AMS **48**, 567 (2001);
H.W. Woodin, *The Continuum Hypothesis, Part II*. Notices of the AMS **48**, 681 (2001).
- [7] F. Hausdorff, *Set Theory*. Translated by J.R. Aumann. American Mathematical Society (2005).